EXERCISE – V | JEE PROBLEMS

1. (a) The incentre of the triangle with vertices $(1, \sqrt{3})$, (0, 0) and (2, 0) is **[JEE 2000(Scr.), 1 + 1]**

(A) $\left(1, \frac{\sqrt{3}}{2}\right)$ (B) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$ (C) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (D) $\left(1, \frac{1}{\sqrt{3}}\right)$

(b) Let PS be the median of the triangle with vertices, P(2, 2), Q(6, –1) and R(7, 3). The equation of the line passing through (1, –1) and parallel to PS is

(A) 2x – 9y – 7 = 0 (B) 2x – 9y – 11 = 0

(C) 2x + 9y – 11 = 0 (D) 2x + 9y + 7 = 0

(c) For points $P(x_1, y_1)$ and $Q(x_2, y_2)$ of the co-ordinate plane, a new distance d(P, Q) is defined by $d(P, Q)=|x_1-x_2| + |y_1 - y_2|$. Let O(0, 0) and A(3, 2). Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from O and A consists of the union of a line segment of finite length and an infinite ray. Sketch this set in a labelled diagram. **[JEE 2000(Mains), 10]**

2. Find the position of point (4, 1) after it undergoes the following transformations successively.

(i) Reflection about the line, y = x – 1

(ii) Translation by one unit along x–axis in the positive direction.

(iii) Rotation through an angle $\pi/4$ about the origin in the anti-clockwise direction. **[REE 2000(Mains), 3]**

3. (a) Area of the parallelogram formed by the lines y = mx, y = mx + 1, y = nx and y = nx + 1 equals

(A) $\frac{|m+n|}{(m-n)^2}$ (B) $\frac{2}{|m+n|}$ (C) $\frac{1}{|m+n|}$ (D) $\frac{1}{|m-n|}$

(b) The number of integer values of m, for which the x co-ordinate of the point of intersection of the lines 3x + 4y = 9 and y = mx + 1 is also an integer, is **[JEE 2001(Scr.)]**

(A) 2 (B) 0 (C) 4 (D) 1

4. (a) Let P(–1, 0), Q(0, 0) and $R(3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is

(A) $\frac{\sqrt{3}}{2}x + y = 0$ (B) $x + \sqrt{3}y = 0$

(C) $\sqrt{3}x + y = 0$ (D) $x + \frac{\sqrt{3}}{2}y = 0$

(b) A straight line through the origin O meets the parallel lines 4x + 2y = 9 and 2x + y + 6 = 0 at points P and Q respectively. Then the point O divides the segment PQ in the ratio

(A) 1 : 2 (B) 3 : 4 (C) 2 : 1 (D) 4 : 3

(c) The area bounded by the curves y = | x | – 1 and y = – | x | + 1 is **[JEE 2002(Scr.)]**

(A) 1 (B) 2 (C) $2\sqrt{2}$ (D) 4

(d) A straight line L through the origin meets the line x + y = 1 and x + y = 3 at P and Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn, parallel to 2x – y = 5 and 3x + y = 5 respectively. Lines L_1 and L_2 intersect at R. Show that the locus of R, as L varies, is a straight line. **[JEE 2002 (Mains)]**

(e) A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinates axes at points P and Q. Find the absolute minimum value of OP + OQ, as L varies, where O is the origin. **[JEE 2002 (Mains), 5]**

5. The area bounded by the angle bisectors of the lines $x^2 - y^2 + 2y = 1$ and the line x + y = 3, is

(A) 2 (B) 3 (C) 4 (D) 6 **[JEE 2004 (Scr.)]**

6. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h,k) with the lines y = x and x + y = 2 is $4h^2$. Find the locus of the point P. **[JEE 2005(Mains), 2]**

7. (a) Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are **[JEE 2007, 3 + 3]**

(A) (4/3, 3) (B) (3, 2/3) (C) (3, 4/3) (D) (4/3, 2/3)

(b) Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1: The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

because

Statement-2: In any triangle, bisector of an angle divides the triangle into two similar triangles.

(A) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1

(B) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1

(C) Statement-1 is true, statement-2 is false

(D) Statement-1 is false, statement-2 is true

8. Consider the lines given by **[JEE 2008, 6]**

$L_1 = x + 3y - 5 = 0$

$L_2 = 3x - ky - 1 = 0$

$L_3 = 5x + 2y - 12 = 0$

Match the statements/Expression in Column-I with the statements/Expressions in Column-II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in OMR.

Column–I	**Column–II**
(A) L_1, L_2, L_3 are concurrent, if	(P) $k = -9$
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(Q) $k = -\frac{6}{5}$
(C) L_1, L_2, L_3 form a triangle, if	(R) $k = \frac{5}{6}$
(D) L_1, L_2, L_3 do not form a triangle, if	(S) $k = 5$

9. The locus of the orthocentre of the triangle formed by the lines **[JEE 2009, 3]**

$(1 + p)\,x - py + p(1 + p) = 0,$

$(1 + q)\,x - qy + q(1 + q) = 0$

and $y = 0$, where $p \neq q$, is

(A) a hyperbola (B) a parabola

(C) an ellipse (D) a straight line

10. A straight line L through the point (3, –2) is inclined at an angle 60º to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$